Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modeling
IE 300

Semester Paper
Spring 2004

Amara Nance

20040920 107

Spring 2004 Research

Basic Queueing Theory

At the beginning of the semester, I started going through a very basic introduction to queueing theory. I have a basic understanding of how queueing works but I have not had enough exposure to this information to do anything with the theory. I learned enough to recognize queueing but nothing farther. I will have to work more with this information to be able to understand queueing theory.

Fitting Model Parameters Using SCoP

My focus of the semester was on fitting parameters to aphid population models. I started with a deterministic model, figuring out how the data was fit without SCoP, and then re-creating the fit that was obtained using SCoP. Then I fit stochastic models that cannot be fit analytically using SCoP.

The first task I had to undertake was to learn how to use the SCoP program. Learning the basics in SCoP was not very difficult, but it took a lot of practice to be able to fit models well quickly. I wrote a tutorial about how to fit data to a model at the end of the semester. See Appendix 1.

After learning how to use SCoP, I used it to fit the same data that was fit analytically. After I obtained the same results as the analytical fit, I was confident in my ability to use SCoP to fit other models.

The models involved a function of the number of current aphids and the number of cumulative aphids and the birth and death rates. Using SCoP, the birth and death rate parameters were easily obtained.

I started fitting data that was the weekly count of adult and nymph aphids at Texas A&M University. The numbers were much larger than the data I fit previously so I ran into problems with the error function being very large. The reason for this is that the least squares error function is used and since I had larger numbers, the differenced in observed and predicted numbers of aphids was much larger.

I tried many models other than the basic, deterministic model for the aphid population growth but I ended up focusing on five models. None of the models that were alternatives to the deterministic model was fitting the new data very well. At this point, I decided to investigate the data.

I graphed the data for each plot and discovered that there was a large amount of variation between plots and between the trees in the plots. Three different patterns emerged when the population data for the plots was graphed individually. There was one type that had a very small amount of aphids with a smooth growth and a small population spike, another type that had a larger spike in population but the growth and decline from that maximum was pretty steady and a group that had a small amount of aphids followed by a huge, sudden growth in population that declined sharply.

I discovered that for the two plots that had a uniform shape in the population, the basic model fit best. For the plots that had the sharp peak in population, the Delayed Super Model fit the best.

After studying this model for the majority of the semester, an alternative model was suggested where the model used the number of live aphids and the number of dead aphids instead of current and cumulate counts of aphids. I tried many different models using this theory but none of the models fit the data well.

When I had exhausted all possibilities I could see using SCoP, I used Mathematica to recreate the results that Prajneshu published in the paper "A Nonlinear Statistical Model for Aphid Population Growth". The research I worked on this semester was based on this paper.

I learned a couple of software programs that are very useful in stochastic modeling, I also learned how much work, and imagination goes into modeling. I hope to continue learning and applying what I learned this semester to more research projects.

Fitting Differential Equations Using Simulation Control Program (SCoP)

Amara Nance
May 17, 2004

Steps in Building a SCoP Program
1. Prepare Model Equations.
2. Make a list of the constants, parameters and variables for the model.
3. Create a model source file containing the equations and lists of variables and parameters.
4. Add to the source file a description of the external conditions applied to the system (initial conditions).
5. Create the SCoP and SCoPfit programs from the source file.

Using SCoP and SCoPfit
1. Run the SCoP Simulation Program.
2. Run the SCoPfit Program.

Model Equations
For the Prajneshu problem, the equations were already generated. The basic model equations are:

$$N' = \lambda N - \mu NY$$

$$Y' = N$$

Where N = current count of aphids and Y = cumulative count of aphids.

List Constants, Parameters and Variables
1. In SCoP, symbols are not used to represent variables in equations. The model source file is a text file so names representing the symbols are used.
2. The names can be from 1 to 20 characters long and can include alphabetic and numeric characters and the under bar. For example, for N in the model equation you can use the name CurrentAphids.
3. The first character of a name must be a letter.
4. Upper case and lower case letters are different.
5. A name cannot contain a space.

Types of Variables
When listing the variables, constants and parameters in the source file, they are separated into different types of variables that are listed in blocks in the source file. There are five different types of variables in SCoP.

1. STATE

> State variables are the variables that require numerical solver algorithms; they are usually the variables that represent the experimental measurements. In the Aphid model, the state variables are the Current Aphids and Cumulative Aphids. The value for state variables is a range with upper and lower limits.

2. ASSIGNED

> Assigned variables require calculations but the values for these variables come directly from explicit equations. There are not any assigned variables in the Aphid model. The value for assigned variables is a range with upper and lower limits.

3. INDEPENDENT

The independent variable is varied during the simulation or it takes its course. In the Aphid model and in many other models, time is the independent variable. When listing the independent variable, you must define an interval over which the variable is valid. You also specify how many intervals there will be in your model. SCoP will perform calculations at each interval. See the example model definition file for examples.

4. CONSTANT

Constants have assigned values that don't need to be calculated or changed in the simulation.

5. PARAMETER

A parameter variable remains fixed during the simulation but can be changed between simulations. The parameter must be assigned a numeric value. The value doesn't have to be correct at this stage since it is easy to change the value in the simulation program.

Variable List for the Basic Aphid Population Model

Type and Description	Name	Value	Units
STATE			
Current Aphid Count	CurrentAphids	FROM 0 TO 3000	Aphids
Cumulative Aphid Count	CumulativeAphids	FROM 0 TO 3000	Aphids
INDEPENDENT			
Time	time	FROM 0 TO 15 WITH 15	Weeks
PARAMETER			
Aphid Birth Rate	BirthRate	1	1/week
Aphid Death Rate	DeathRate	.001	1/week

Create the Model Source File

After the variable list and model equations are created, you create the model source file in the SCoP model editor. SCoP has its own language that you have to follow when creating the model source file. See the example model source file.

1. The source file is divided into blocks that are indicated by braces {}. You have to list the definition (variable) blocks first, followed by the equation blocks.
2. The key words such as variable type, equation type and commands in the SCoP language are written in all capital letters.
3. The source file has to have a file name with the extension **.mod**.

4. When creating the block for the model equations, the block name corresponds to the type of equations that you use for your model. In the basic aphid population model, the equations are ordinary differential equations so they are in a block titled DERIVATIVE. There are four types of equations that SCoP can solve. The block titles are:
   DERIVATIVE
   LINEAR
   NONLINEAR
   KINETIC
5. When using the DERIVATIVE name for the equation block, you must use the following rules:
   o Each differential equation must be rearranged to have the highest derivative isolated on the left-hand side of the equation.
   o A prime (') symbol is used to represent a derivative with respect to the independent variable.
   o In addition to the DERIVATIVE name in the block, the block is given an additional name such as aphids in the basic aphid population model. This name is used when writing the command for SCoP to solve the equations.
6. Now, you have to create the BREAKPOINT block. The BREAKPOINT block controls what happens in the simulation at every output breakpoint that you defined in the INDEPENDENT variable block.

Adding External Conditions to the Model Source File
- You must enter initial conditions for each STATE variable to begin the solution.
1. Create a block with the title INITIAL and enter initial conditions for each STATE variable.

Final Source File - Basic Aphid Population Model
In addition to the blocks already created, you can include additional information about
the model after the TITLE block by typing a colon : then the information.

TITLE Aphid Population Model
:Basic aphid population model from Prajneshu's paper "A Non-linear Statistical Model
:for Aphid Population Growth

PARAMETER {
BirthRate = 1
DeathRate = .001
}

INDEPENDENT {
time   FROM 0 TO 15 WITH 15   (weeks)
}

PLOT CurrentAphids, CumulativeAphids VS time

STATE {
CurrentAphids   FROM 0 TO 3000
CumulativeAphids   FROM 0 TO 3000
}

INITIAL {
CurrentAphids=1
CumulativeAphids=1
}

DERIVATIVE aphids {
CurrentAphids' = BirthRate * CurrentAphids - DeathRate * CurrentAphids *
CumulativeAphids
CumulativeAphids' = BirthRate * CurrentAphids
}

BREAKPOINT {SOLVE aphids}

Saving the Model Source File
1. After the model source file is complete, save the file as your model name
   followed by the **.mod** extension. For the basic aphid population model the source
   file name is Basic.mod.

Create SCoP and SCoPfit Programs

1. After the source model has been saved, click Run in the SCoP model editor then click Compile Only. This will create the programs and files needed to run the SCoP simulation program and the SCoPfit program.

Running a SCoP Simulation

- Before using SCoPfit, you have to run a simulation so that you can create the reference file and so SCoPfit will save the parameters that are generated when running the optimization in SCoPfit. If you do not run the simulation before you run SCoPfit, the parameters you obtain in SCoPfit might not be saved. This is a glitch in the SCoPfit program.
1. From the SCoP model editor, click Run then Simulate.
2. The SCoP simulation program will start, press enter to continue.
3. You will be prompted to enter a variable definition file. SCoP generates a variable file with the same name as your model with a **.var** file extension. Pressing enter at this prompt will use the default variable file. You are now at the SCoP simulation main menu.
- From the main menu, you can modify output format, parameter values, get plotted or tabular output and or select another option that is available.
4. To modify variable values, type E to get to the variable editor. Before you get to the editor, a screen comes up that has all the commands used in the editor. Type X to exit the editor.
5. To create the reference file, type O to get to the SCoP output format menu. For the aphid population model, CurrentAphids is the variable that we have data for so we want the output to have the calculations of the number of current aphids vs. time.
6. From the output format menu, type V then enter to select the vertical axis variables.
7. Type the variable name CurrentAphids then enter. It will ask if the variable name CurrentAphids is correct, type y then enter.
8. From the output menu, type x then enter to get back to the main menu.
9. When you run the simulation, you have a choice between tabular and plotted output. Type P then enter for plotted output or T then enter for tabular output.
10. After the simulation is complete, for plotted output, type W to write the output to a file.
11. You can also press enter and you are at the SCoP post-simulation menu. Type W then enter and you will be prompted for a file name. Type your file name followed by an **.out** extension. An example is Basic.out. You will be prompted to enter a label for the data. You can either create a label for your data or leave it blank. Press enter to continue. The simulation output will be written to the file you specified.
12. Exit the post-simulation menu and then exit SCoP.
13. The output file is saved in the folder where your model source file is located. It can be opened using Notepad.

Preparing the Reference File
1. Open the output file created in the simulation using Notepad.
2. Edit the file by replacing the data SCoP generated with your data. You have to edit each entry, it does not work if you just copy and paste the data.
3. Save the output file with your data. Be sure to include the **.out** extension.

Running SCoPfit
1. From the SCoP model editor, click Run then Fit.
2. Press enter to begin.
3. You will be prompted to specify a control file name, SCoP fit will specify a name, you can either use that name or rename the file, be sure to include the .ctl file extension. Press enter.
4. You will get a notice that you have to specify at least two parameters to fit, press enter; this will get taken care of in another step.
5. When you get to the SCoPfit main menu, type c then enter to modify optimization control parameters. This is where you will specify the variable file, reference file and parameters to be optimized.
6. You are now at the optimization control menu. There are several steps that need to be completed through this menu before you start the optimization procedure.
   o Type v to select a variable definition file. The variable definition file was created when you complied the model from the SCoP model editor. The name of the variable definition file is the name of your model file with the file extension .var. i.e. basic.var. Press enter. You should be back at the optimization control menu.
   o Type R and enter to select a reference data file. This file is the reference file you created earlier. Type the name in and press enter.
   o From the optimization control menu, type P then enter to select the optimization parameters. You will see a screen with keyboard commands for the parameter editor. Press enter to continue. When you get to the screen where your variables are listed, use enter to get to the variables you want to optimize and type f in front of the variables. Press x then enter to get back to the optimization control menu. Then exit to the SCoP fit main menu using x.
7. From the SCoPfit main menu, you can use e to check the error function to see how close your initial parameters are to the data. Use x to exit from this window and return to the main menu.
8. Now, you are ready to run the optimization. To get the best fit, you must fit the parameters iteratively.
   o Type O to perform an optimization.
   o You will be prompted: Do you want smooth curves drawn through the output points? Type y or n
   o Now there is a graph with your data points and the curve that results from the parameters that were fit. In the first iteration, the fit is not always very good.
   o Press enter to view the error function value. This is the least squares error function. You will be prompted to view and save the parameters here

press y, enter to view the parameters then y, enter again to save them. To iteratively fit the parameters, you must save the parameters. They will be saved to the variable definition file.

o After you save the parameters, you will return to the main menu.

o To get a better fit, you can iteratively reduce the error tolerance and/or the step size.

- Reducing the error tolerance usually has a greater effect on the quality of fit. The error tolerance adjustment in the optimization control menu.
- From the main menu, type c to get to the optimization control menu.
- From the optimization control menu, type t to specify the error tolerance. The default value is 0.1. Reduce the error tolerance by small amounts (i.e. 0.1) and run the simulation again.

o Iteratively run the optimization and reduce the error tolerance until the error function is minimized.

Using SCoP fit to calculate statistics of fit

SCoP fit will calculate the statistics of the fit you obtain by running optimizations.

1. From the main menu, type w to select a file to save the statistics of the fit. Type the file name, including a .sta file extension to indicate it is a statistics file.
2. From the main menu, type s to go to the SCoP fit statistics menu.
3. From the statistics menu, select which statistics you would like to calculate. The statistics will automatically be saved to your statistics file that you selected earlier.

Additional Information

- When fitting the model in SCoPfit, if you get the error message:

    "The increment in the independent variable is less than machine roundoff error. Press <Enter> to abort the run."

    Press enter to continue.

    First try to increase the error tolerance to something larger than the default value of 0.1. Start with an error tolerance of 1.0 and run the optimization again. Continue to fit the model using the iterative process described in "Running SCoPfit" but decrease the error tolerance by very small amounts so you avoid getting the error.

    You can also try changing the initial values of your state variables to see if you can avoid getting this error. If these things do not work, you should probably change your model.

- SCoPfit also has different integrators that can be specified to solve the model equations for the STATE variables. To use a different integrator, you must specify which integrator you want to use in the BREAKPOINT block of the model source file. After the commands in the BREAKPOINT block, type METHOD then the integrator you would like to use. For example if you want to use the euler integrator your BREAKPOINT block would be:

    BREAKPOINT {SOLVE aphids METHOD euler}

- There are 6 different integrators that can be used in SCoP when the model equations are ordinary differential equations:

1. adrunge
   This is the default integrator that SCoP uses. It is a fourth-order Runge-Kutta method with automatic adjustment of the time increment to a specified accuracy of the solution.
2. euler
   This is the simplest method used. It is slower than the default integrator so fitting using this integrator takes much longer.
3. adeuler
   The same as euler but with automatic adjustment of the time increment to a specified accuracy of the solution.
4. heun
   Adds a corrector step to the euler method.
5. adams
   Uses an Adams-Bashforth predictor, an Adams-Moulton modifier and corrector iteration.
6. clsoda
   An improved Gear method for stiff sets of equations. Automatically switches to an Adams method when the equations are not stiff. Uses a variable time increment and multiple function evaluations.

Basic Model
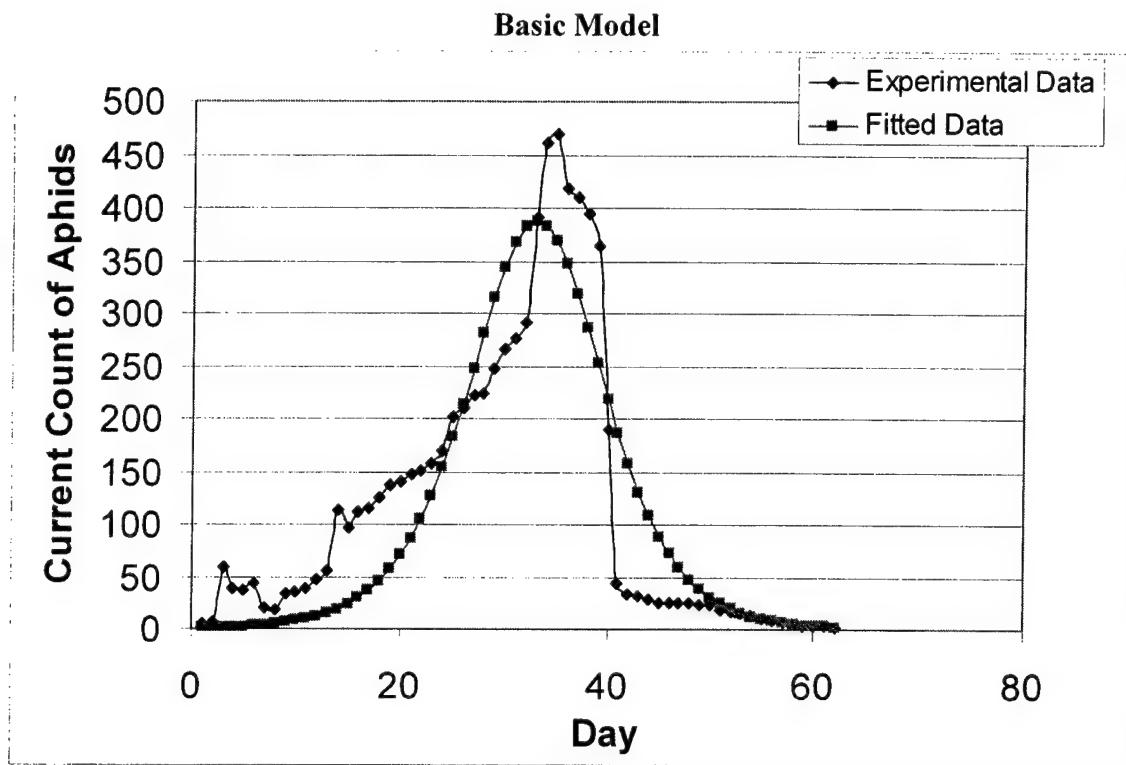
Statistics of the Fit for
Aphid Population Model

Error function = 59.340831
BirthRate = 0.228864
DeathRate = 0.000068

Serial Correlation Coefficient for Set Number: 1

CurrentAphids: rho = 0.8473   t(59 d.f.) = 12.3599   t(0.05) = 1.6712
Covariance Matrix
  1.27260e-05 1.21537e-09
  1.21537e-09 7.92782e-12
Correlation Matrix
 1.00000e+00 1.21000e-01
  1.21000e-01 1.00000e+00
Standard Deviations (S), 95% Confidence Limits (L), and t-statistics (t) of Parameters

Expected value of t = 2.00105

S = 3.56736e-03, L = 7.13846e-03, t(0.05) = 64.15507
S = 2.81564e-06, L = 5.63424e-06, t(0.05) = 24.01721

Delayed Feedback Model
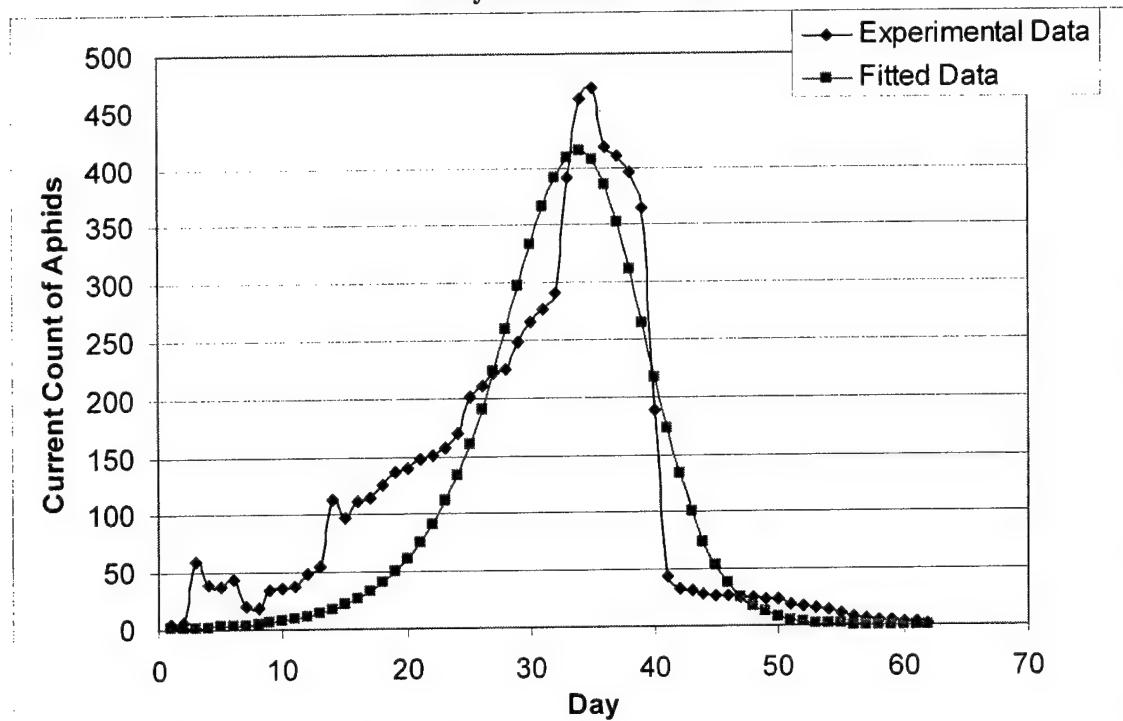
Statistics of the Fit for
 Aphid Population Model

Error function = 53.099784
BirthRate = 0.220481
DeathRate = 0.000470

Serial Correlation Coefficient for Set Number: 1

CurrentAphids: rho = 0.8200   t(59 d.f.) = 11.0956   t(0.05) = 1.6712
Covariance Matrix
 7.14248e-06 -1.61485e-08
 -1.61485e-08 3.13035e-10
Correlation Matrix
 1.00000e+00 -3.41517e-01
 -3.41517e-01 1.00000e+00
Standard Deviations (S), 95% Confidence Limits (L), and t-statistics (t) of Parameters

Expected value of t = 2.00105

S = 2.67254e-03, L = 5.34789e-03, t(0.05) = 82.49862
S = 1.76928e-05, L = 3.54042e-05, t(0.05) = 26.56198

Super Feedback Model
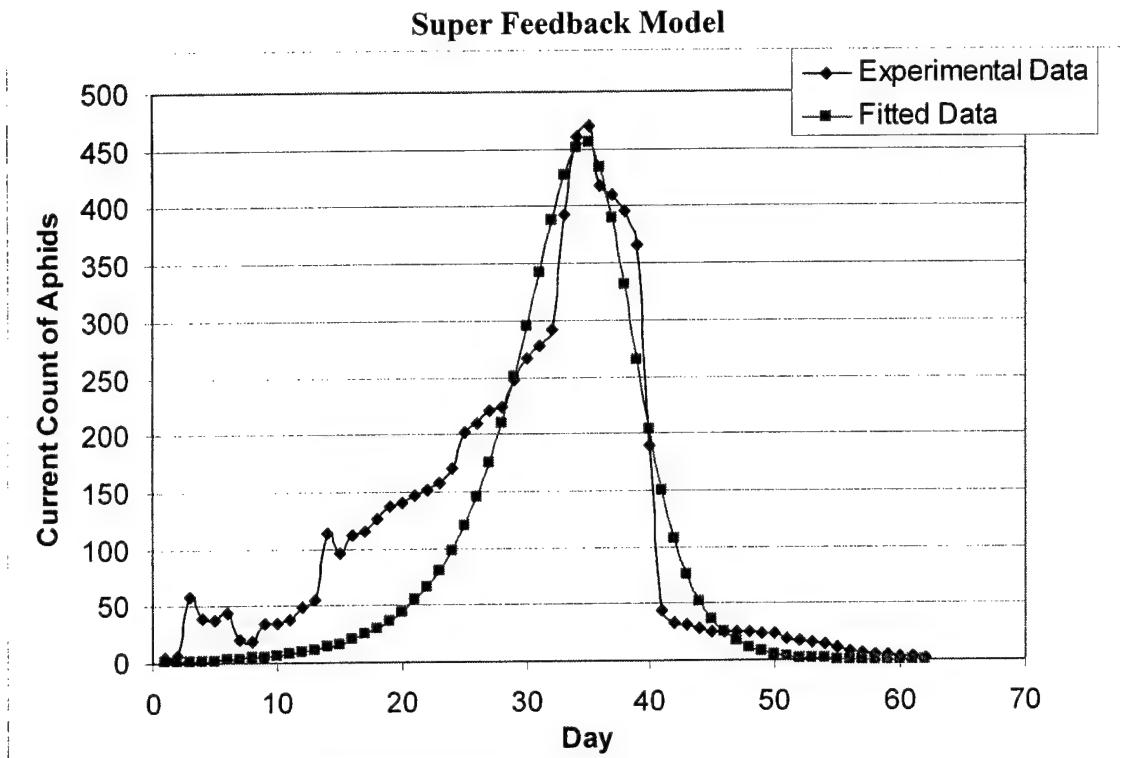
Statistics of the Fit for
Aphid Population Model

Error function = 53.056811
BirthRate = 0.200048
DeathRate = 0.000000

Serial Correlation Coefficient for Set Number: 1

CurrentAphids: rho = 0.8215   t(59 d.f.) = 11.1611   t(0.05) = 1.6712
Covariance Matrix
 4.86741e-06 -4.53059e-11
 -4.53059e-11 1.20751e-15
Correlation Matrix
 1.00000e+00 -5.90964e-01
 -5.90964e-01 1.00000e+00
Standard Deviations (S), 95% Confidence Limits (L), and t-statistics (t) of Parameters

Expected value of t = 2.00105

S = 2.20622e-03, L = 4.41476e-03, t(0.05) = 90.67456
S = 3.47492e-08, L = 6.95349e-08, t(0.05) = 12.26124

Modified Super Feedback Model
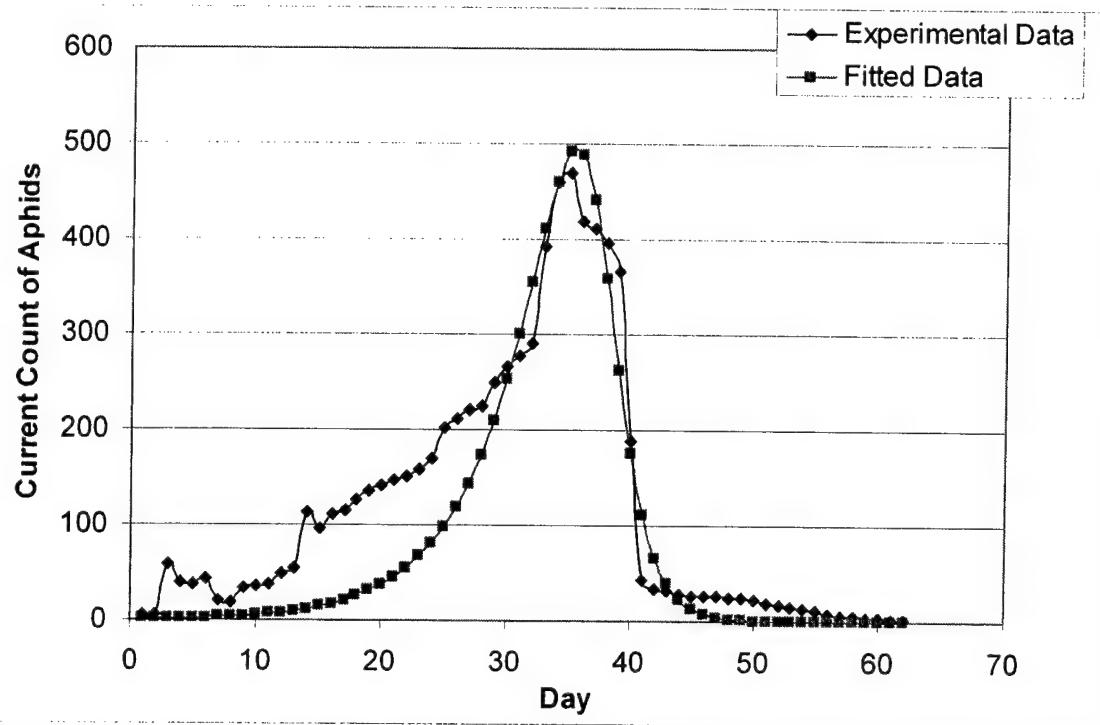
Statistics of the Fit for
 Aphid Population Model

Error function = 54.557056
BirthRate = 0.191355
DeathRate = 0.000000

Serial Correlation Coefficient for Set Number: 1

CurrentAphids: rho = 0.8152   t(59 d.f.) = 10.9016   t(0.05) = 1.6712
Covariance Matrix
 3.83828e-06 -1.21920e-13
 -1.21920e-13 7.67721e-21
Correlation Matrix
 1.00000e+00 -7.10237e-01
 -7.10237e-01 1.00000e+00
Standard Deviations (S), 95% Confidence Limits (L), and t-statistics (t) of Parameters

Expected value of t = 2.00105

S = 1.95915e-03, L = 3.92036e-03, t(0.05) = 97.67236
S = 8.76197e-11, L = 1.75332e-10, t(0.05) = 7.57117

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modeling

Printed Name and Signature _Amara Nance_ *Umara Nance*

For the week ending _2/27/04_____

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
I used the basic model to fit new, weekly data.

Key Research Findings:
The basic model fits this data well. The value of the error function is still high for the total and nymph population but low for the adult population.

Basic Model- Adults
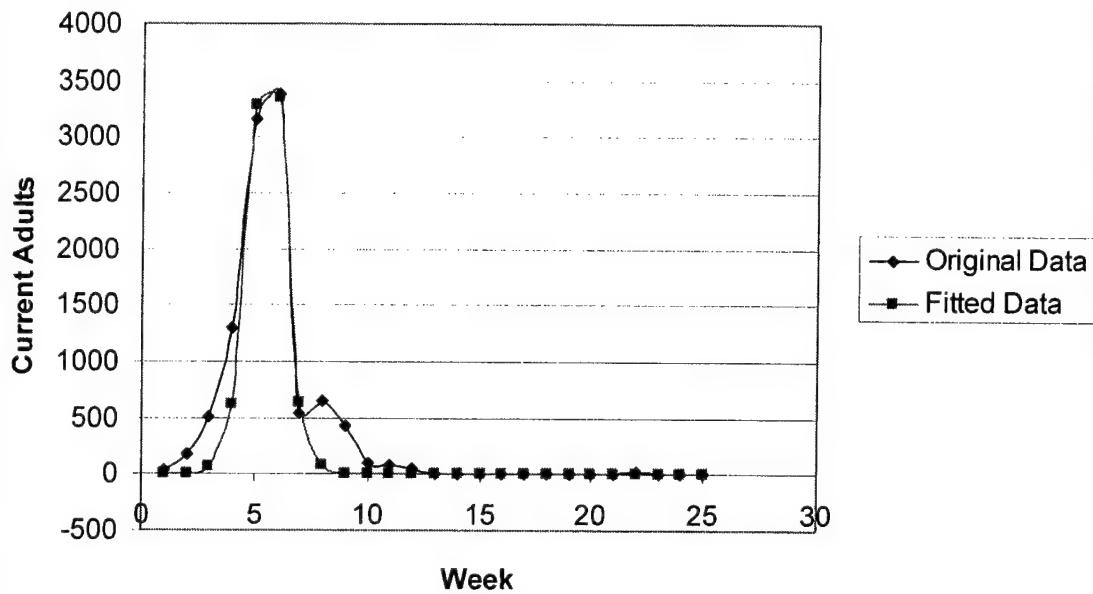
Statistics of the Fit for
Aphid Population Model for Total Adult Aphids

Error function = 230.381518
BirthRate = 2.168350
DeathRate = 0.000535

Serial Correlation Coefficient for Set Number: 1

CurrentAphids: rho = 0.2899   t(22 d.f.) = 1.4525   t(0.05) = 1.7170
Covariance Matrix
 6.03859e-04 -9.08469e-08
 -9.08469e-08 5.48128e-10
Correlation Matrix
 1.00000e+00 -1.57907e-01
 -1.57907e-01 1.00000e+00
Standard Deviations (S), 95% Confidence Limits (L), and t-statistics (t) of Parameters

Expected value of t = 2.07400

S = 2.45735e-02, L = 5.09655e-02, t(0.05) = 88.23925
S = 2.34121e-05, L = 4.85568e-05, t(0.05) = 22.85357

Basic Model- Nymphs
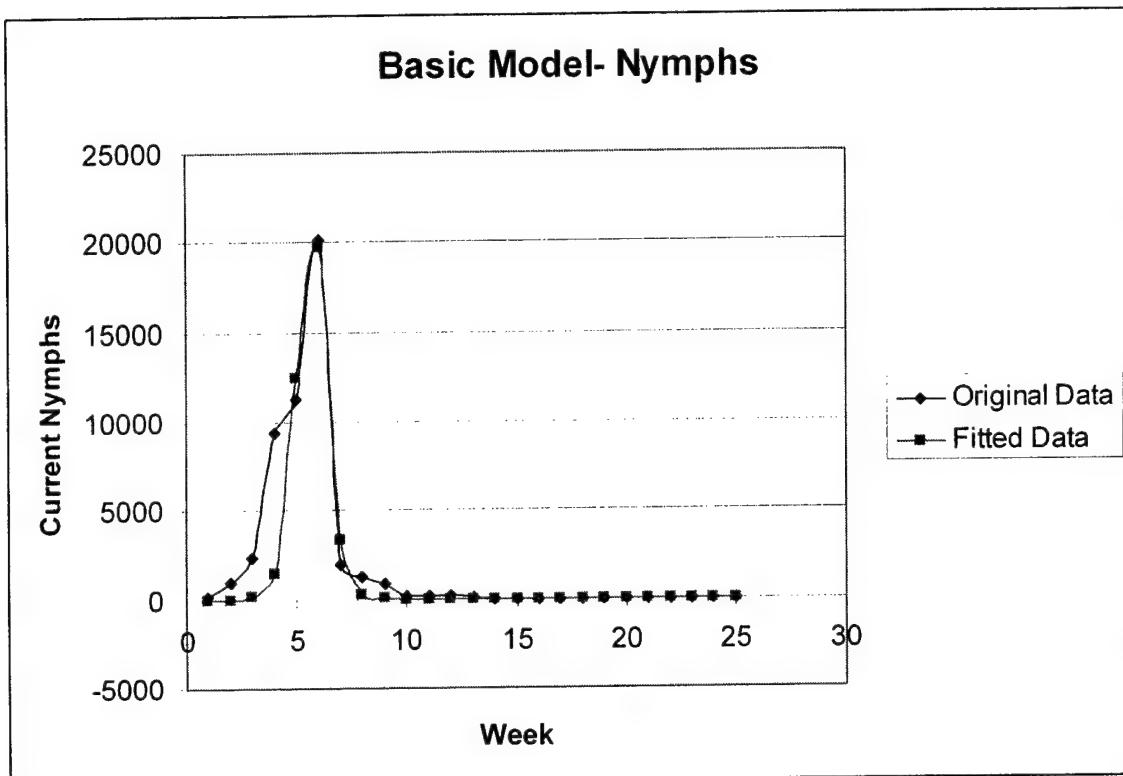
Statistics of the Fit for
 Aphid Population Model

Error function = 1780.362412
BirthRate = 2.446178
DeathRate = 0.000130

Serial Correlation Coefficient for Set Number: 1

CurrentAphids: rho = 0.0499   t(22 d.f.) = 0.2396   t(0.05) = 1.7170
Covariance Matrix
  1.83882e-03 -1.17251e-07
 -1.17251e-07 7.61736e-11
Correlation Matrix
  1.00000e+00 -3.13289e-01
 -3.13289e-01 1.00000e+00
Standard Deviations (S), 95% Confidence Limits (L), and t-statistics (t) of Parameters

Expected value of t = 2.07400

S = 4.28815e-02, L = 8.89362e-02, t(0.05) = 57.04512
S = 8.72775e-06, L = 1.81014e-05, t(0.05) = 14.89353

Basic Model- Total
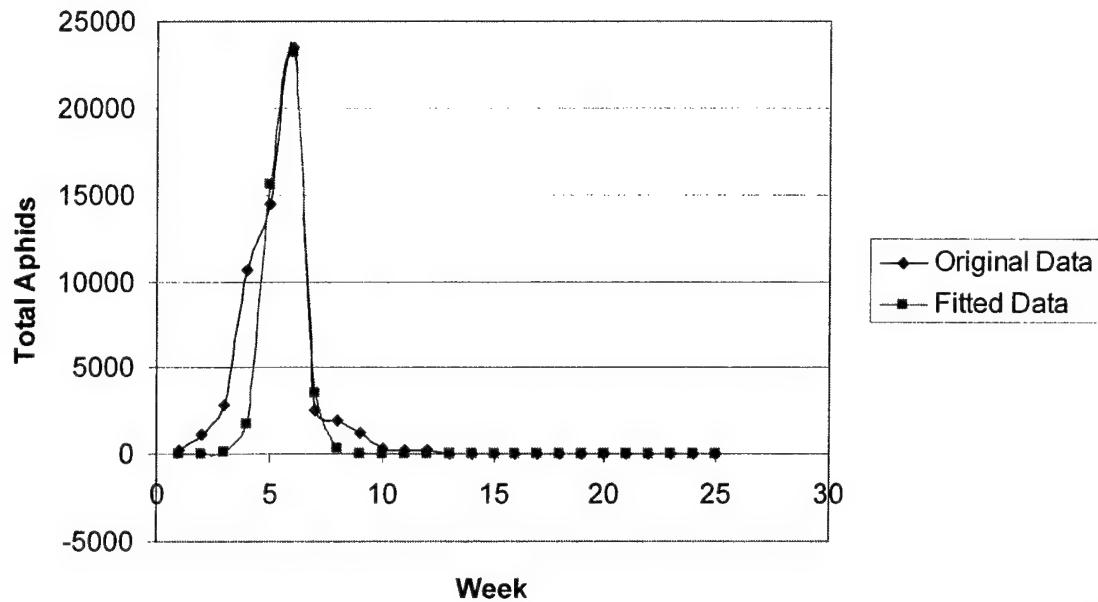
Statistics of the Fit for
 Aphid Population Model

Error function = 2024.040115
BirthRate = 2.503990
DeathRate = 0.000112

Serial Correlation Coefficient for Set Number: 1

CurrentAphids: rho = 0.0996   t(22 d.f.) = 0.4798   t(0.05) = 1.7170
Covariance Matrix
  1.55573e-03 -9.74972e-08
 -9.74972e-08 5.11921e-11
Correlation Matrix
  1.00000e+00 -3.45480e-01
 -3.45480e-01 1.00000e+00
Standard Deviations (S), 95% Confidence Limits (L), and t-statistics (t) of Parameters

Expected value of t = 2.07400

S = 3.94428e-02, L = 8.18043e-02, t(0.05) = 63.48412
S = 7.15487e-06, L = 1.48392e-05, t(0.05) = 15.59316

Original Data		Texas A&M Parameters		NMSU Parameters		Scop Paramet	
1	1	1	0.11	1	0.110149	1	
2	18	2	0.5799	2	0.582048	2	4.5
3	20	3	3.041854	3	3.060084	3	19.
4	60	4	15.5445	4	15.66701	4	68.
5	121	5	69.74753	5	70.27485	5	11!
6	67	6	184.754	6	184.6256	6	66.
7	4	7	151.2754	7	148.7707	7	19.
8	15	8	45.28102	8	44.07649	8	4.3
9	10	9	9.479728	9	9.185505	9	0.9
10	1	10	1.831427	10	1.769543	10	0.2
11	0	11	0.348283	11	0.335688	11	0.0
12	1	12	0.066034	12	0.063495	12	0.0

Scop Simulation Data Using Prajneshu's Results
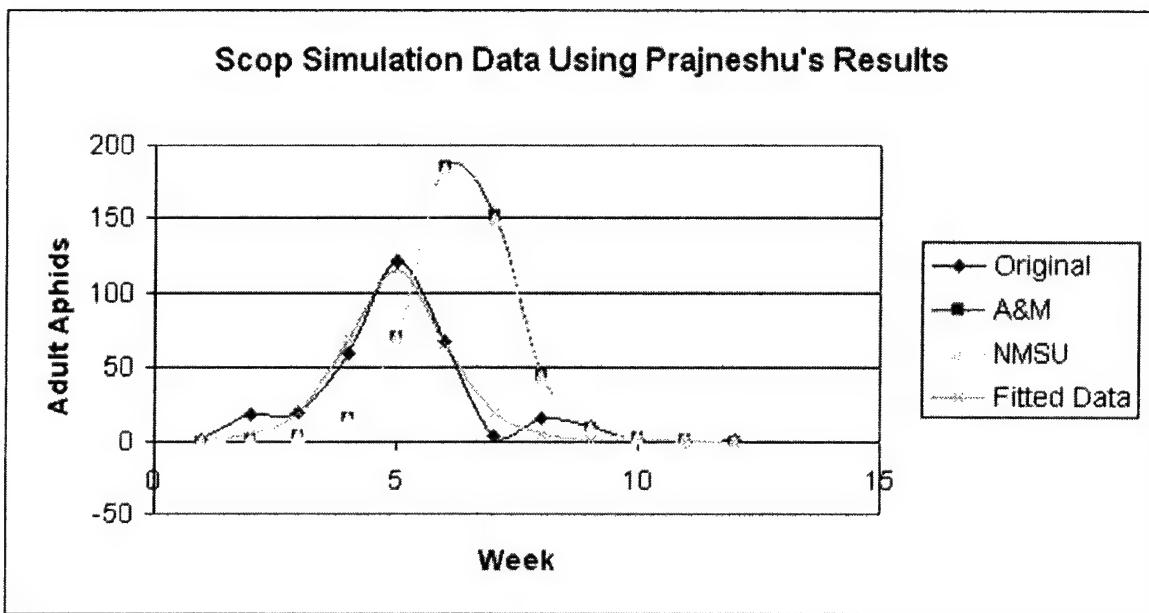
	a	b	d	c	N	r	$\lambda=r$	$\mu=(c\lambda)^{-1}$
	2100000	1.67	4364	86.27235	0.110218	1.669235	1.669235	0.006944
my numbers	0.110199	1.67318	-0.000229	85.89822	0.110149	1.672413	1.672413	0.006961

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature _Amara Nance _Amara Nance

For the week ending _3/5/04_____

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
I tried using different integrators in Scop to see if an alternate integrator would produce better results.
I fit a 3 Parameter model:
$X' = \lambda X - \mu XY2$
$Y1' = \lambda X - \gamma Y1$
$Y2' = \gamma Y1$
I also tried to create an alternate model.
I started fitting models to a reduced data set.

Key Research Findings:
The other integrators in Scop come up with the same result as the default integrator but are slower most of the time. The default integrator is the best for the models that I am working with now.
The three parameter model fits the data almost as well as the Basic Model. I am still working on variations to get it to fit better.
I don't have any results of the reduced data set model fitting yet.

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature ___Amara Nance _Amara Nance_

For the week ending _3/12/04_

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
I fit the existing models to the reduced data set (15 weeks of data). I also used the adeuler integrator to fit the 3 parameter model. I also went through and learned how ScoP is fitting the models by reading the Numerical Analysis book.

Key Research Findings:
With the reduced data set, the basic model still fits the best. The three parameter model would start to fit then the error function would go to infinity. I used the adeuler integrator and I got the model to fit. The fit I got for the 3 parameter model is only slightly better than the delayed super feedback model which is the worst of the models I was working with previously.

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature ___Amara Nance *Amara Nance*
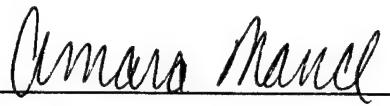
For the week ending _3/19/04_____

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
I worked on grouping and analysis of the data to look for variation in the observations. I graphed total, adult and nymph aphids by tree. I also graphed each plot of trees individually.

Key Research Findings:
The total number of aphids from all trees had tremendous variability. There were some trees that had up to 1000 aphids one week and other trees that didn't have any aphids that same week. When I broke the data into graphs of each plot, there were three main shapes that the graphs took. One type had a huge spike in either week 5 or 6 and then dropped sharply. The other type of graph was pretty uniform, it had the spike but all of the trees seemed to have the same type of behavior. The third type of graph had trees with relatively few aphids and there was much more variability in the number of aphids for the trees.

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature ___Amara Nance *Amara Nance*

For the week ending _4/02/04_____

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
I picked three representative clusters of trees from the graphs of the plots. I have one representing the high number of aphids (Plot 9, Cluster 1), one for the medium number of aphids (Plot 4, Cluster 1) and one representing the low number of aphids (Plot 12 Cluster 4)

Key Research Findings:
For the high level of aphids, the Delayed Super Feedback Model with the squared term $(x'=\lambda x + \mu xy2^2)$ fit the best. For the low level of aphids, the basic model fit the best but the 3 parameter model was very close. For the medium level of aphids, the super feedback model fit the best but all of the more complex models fit better than the basic model

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature ___Amara Nance *Amara Nance*
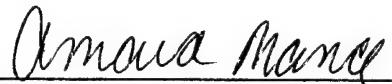
For the week ending _4/09/04_____

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
I used Mathematica to solve the deterministic model. $(N(t) = ae^{-bt}(1+de^{-bt})^{-1})$

Key Research Findings:
The parameters a, b, and d were solved. They are functions of λ, μ and N_0. The transformations from a, b, and d to λ, μ and N_0 are in Prajneshu's paper.

Printed Name and Signature ___Amara Nance *Amara Nance*
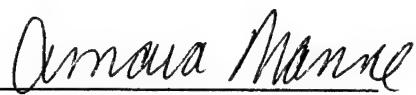
For the week ending _4/16/04_____

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
Transformed the parameters a, b, and D into λ, μ and N_0. Ran a Scop simulation to compare the parameters from the deterministic equation to the actual values.

Key Research Findings:
The parameters did not produce curves that resembled the data. I will check the solution and the transformations to make sure they were done correctly.

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature ___Amara Nance___ *Amara Nance*

For the week ending _4/23/04_____

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
Re-checked deterministic solution and transformations. Also compared my solutions to solutions from A & M and re-ran simulations in Scop for comparison.

Key Research Findings:
I found a problem with my transformation of λ, after that problem was fixed, my values for λ, μ and N_0 were the same as the values they got at A & M however, the values for a, b, and d were different. I ran a simulation with my values as well as with the values from A & M and got the same results. I also fit the data and got $\lambda = 1.54$ and $\mu = .0102182$ using $N_0 = 1$.

From the transformations, the values were:

$\lambda = 1.67$

$\mu = 0.0069$

$N_0 = 0.11$

Investigations into the Application of Cumulant Functions in Operations Research and
Stochastic Modeling

Printed Name and Signature ___Amara Nance *Amara Nance*

For the week ending _4/30/04_____

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of
effort to this research project this week.

Research Activities:
Re-checked deterministic solution and transformations. Also compared my solutions to
solutions from A & M and re-ran simulations in Scop for comparison. Defined the Scop
model to correctly define zero and one for the independent variable time.
Fit models to the number of total aphids in plots to find plots where the alternative
models fit better than the basic model.

Key Research Findings:
The fits from the transformations in Prajneshu's paper are the same for the NMSU
parameters, the A&M parameters and the parameters from Scop Fit.
I am still fitting plots to see if alternative models fit better than the basic model. Right
now it seems that for plots that have a very sharp decline to almost zero aphids, the
alternative models are fitting well. The best model seems to be the Delayed Super
Feedback model. For plots with a more gradual build up and drop off of aphids, the basic
model is fitting better. The plots that I have fit where the basic model fits best are plots 1
and 3. The delayed super feedback model fits best for plots 6 and 9. I will fit other plots
similar to plots 6 and 9 to see if the delayed super feedback model continues to fit better
than the basic model.

Investigations into the Application of Cumulant Functions in Operations Research and
Stochastic Modeling

Printed Name and Signature ___Amara Nance *Amara Nance*

For the week ending _5/7/04_____

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of
effort to this research project this week.

Research Activities:
I started writing a tutorial for SCoP. I'm just covering the fitting of parameters to
differential equations and the processes I have gone through to fit the aphid population
models.

Key Research Findings:

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modeling

Printed Name and Signature ___Amara Nance___ *Amara Nance*
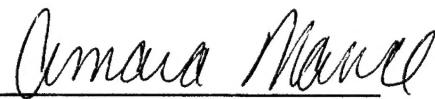
For the week ending _5/14/04_____

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
I finished the SCoP tutorial. I also tried to fit an alternative aphid population model that had x1 = live aphids and x2 = dead aphids.

Key Research Findings:
I fit seven different types of live-dead aphid models to three different sets of data. The model only fit one set of data for one model. It did not fit better than the current-cumulative aphid model. All of the other models did not even approach a fit.

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature _Amara Nance____ *Amara Nance*

For the week ending _2/6/04_____

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
Met with Martin to go over Mathematica program for calculating time in queue.

Key Research Findings:

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature _Amara Nance *Amara Nance*
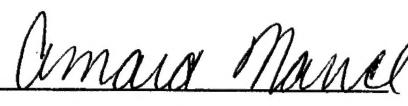
For the week ending _2/13/04

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
Created the basic aphid population model in Scop. Used ScoPfit to fit the basic aphid population model.

Key Research Findings:
By using ScoP fit, I simulated the analytical fitting of the data.

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature _Amara Nance_ *Amara Nance*

For the week ending _2/20/04_

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
I developed and fit data using the Super Feedback and Delayed Feedback models. I also tried to come up with alternative models to fit the data.

Key Research Findings:
The Super and Delayed Feedback models fit the sharp decline in the aphid population well, but fail to capture the gradual increase in population. The alternative models I created either fit the population increase or the sharp decrease but failed to capture both simultaneously.